Graduate Macroeconomic Theory (ECN 200E)
Week 9: New Keynesian Model

Jinho Kim

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1 Demand Block

1.1 Euler Equation → Dynamic IS

Step 1: Euler equation in consumption. The log-linearized household Euler equation is:

$$E_t \Delta \hat{C}_{t+1} = \frac{1}{\sigma}\,\hat{r}_t + \frac{1}{\sigma}\,E_t \Delta \hat{z}_{t+1} \qquad \longleftarrow \qquad \hat{y}_t = \hat{c}_t$$

where $\hat{z}_t$ is an exogenous demand shock (preference shifter) and $\hat{r}_t$ is the real interest rate.

Recall: AR(1) structure of the demand shock $\hat{z}_t$.

$$\hat{z}_{t+1} = \rho_z \hat{z}_t + \varepsilon_{t+1}$$

$$\begin{aligned} \Delta \hat{z}_{t+1} \;&=\; \hat{z}_{t+1} - \hat{z}_t \;=\; (1-\rho_z)\hat{z}_t + \varepsilon_{t+1} \\ E_t \Delta \hat{z}_{t+1} \;&=\; (1-\rho_z)\hat{z}_t, \qquad E_t \hat{z}_{t+1} \;=\; (\rho_z - 1)\hat{z}_t \end{aligned}$$

Step 2: Rewrite using Fisher equation. Using $\hat{\imath}_t = \hat{r}_t + E_t \hat{\pi}_{t+1}$ (Fisher) $\Rightarrow \hat{r}_t = \hat{\imath}_t - E_t \hat{\pi}_{t+1}$, and substituting $\hat{y}_t = \hat{c}_t$:

$$E_t \Delta \hat{y}_{t+1} = \frac{1}{\sigma}\,\hat{r}_t + \frac{1}{\sigma}\,E_t \Delta \hat{z}_{t+1}$$

Expanding $E_t \Delta \hat{y}_{t+1} = E_t \hat{y}_{t+1} - \hat{y}_t$ and solving for $\hat{y}_t$:

$$\hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{\sigma}\big(\hat{\imath}_t - E_t \hat{\pi}_{t+1}\big) + \frac{1}{\sigma}(1-\rho_z)\hat{z}_t$$

Step 3: Define the output gap and subtract natural output. Define:

$$\tilde{y}_t \;=\; \hat{y}_t - \hat{y}_t^n$$

Substituting $\hat{y}_t = \tilde{y}_t + \hat{y}_t^n$ into the equation above:

$$\tilde{y}_t = E_t\big(\hat{y}_{t+1} - \hat{y}_{t+1}^n\big) \; + \; \underbrace{\big(-\hat{y}_t^n + E_t\hat{y}_{t+1}^n\big)}_{E_t\Delta\hat{y}_{t+1}^n} - \frac{1}{\sigma}\big(\hat{\imath}_t - E_t\hat{\pi}_{t+1}\big) \; + \; \frac{1}{\sigma}(1-\rho_z)\hat{z}_t$$

$$= E_t\tilde{y}_{t+1} + E_t\Delta\hat{y}_{t+1}^n - \frac{1}{\sigma}\big(\hat{\imath}_t - E_t\hat{\pi}_{t+1}\big) + \frac{1}{\sigma}(1-\rho_z)\hat{z}_t$$

Step 4: Compute $E_t\Delta\hat{y}_{t+1}^n$. Using $\hat{y}_t^n = \dfrac{1+\varphi}{\sigma+\varphi}\hat{a}_t$ (from the supply block):

$$E_t\Delta\hat{y}_{t+1}^n = \frac{1+\varphi}{\sigma+\varphi}(\rho_a - 1)\hat{a}_t$$

Substituting back and defining the **natural rate of interest** as the exogenous terms:

$$\tilde{y}_t = E_t\tilde{y}_{t+1} - \frac{1}{\sigma}\left[\big(\hat{\imath}_t - E_t\hat{\pi}_{t+1}\big) + \underbrace{\frac{\sigma(1+\varphi)}{\sigma+\varphi}(1-\rho_a)\hat{a}_t - (1-\rho_z)\hat{z}_t}_{-\hat{r}_t^n}\right]$$

where:

$$\hat{r}_t^n \;\equiv\; \frac{\sigma(1+\varphi)}{\sigma+\varphi}(1-\rho_a)\hat{a}_t - (1-\rho_z)\hat{z}_t$$

Dynamic IS (final):

$$\boxed{\tilde{y}_t = E_t\tilde{y}_{t+1} - \frac{1}{\sigma}\Big[(\hat{\imath}_t - E_t\hat{\pi}_{t+1}) - \hat{r}_t^n\Big]} \qquad \text{(Dynamic IS)}$$

2 Supply Block: Intermediate Firm's Problem

2.1 Optimal Price Setting

A firm j re-optimizing in period t chooses $p_t(j)$ to maximize the expected present value of profits *over all future periods for which $p_t(j)$ remains in effect*. The raw objective function is:

$$\max_{p_t(j)} \quad \sum_{s=0}^{\infty} E_t\left(\beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[\frac{p_t(j)}{P_{t+s}}\, y_{t+s}(j) - \phi_{t+s}\, y_{t+s}(j) \right] \right)$$

where $\beta^s \frac{\lambda_{t+s}}{\lambda_t}$ is the stochastic discount factor (SDF) and $p_t(j)$ is set *today* and held fixed for as long as the firm cannot re-optimize.

Decomposing by whether the price is ever reset. Split the full expected sum according to two mutually exclusive events:

$$= \underbrace{\sum_{s=0}^{\infty} \theta^s\, E_t\left(\beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[\frac{p_t(j)}{P_{t+s}}\, y_{t+s}(j) - \phi_{t+s}\, y_{t+s}(j) \right] \right)}_{\textbf{(I)}\,:\ \text{prob. } \theta^s \text{ of } \textit{never} \text{ resetting the price up to } t+s}$$

$$+ \underbrace{(1-\theta) \sum_{h=1}^{\infty} \theta^{s-h} E_t\left(\beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[\frac{p_{t+s}(j)}{P_{t+s}}\, y_{t+s}(j) - \phi_{t+s}\, y_{t+s}(j) \right] \right)}_{\textbf{(II)}\,:\ \text{prob. } (1-\theta)\cdot\theta^{s-h} \text{ of resetting } \textit{at least once} \text{ in the future}}$$

Key insight: In term **(II)**, once the resetting happens the firm will choose a *new* optimal price $p_{t+h}(j)$ at that future date $t+h$. That future re-optimization is therefore *irrelevant to the current choice* of $p_t(j)$, so term **(II)** can be **deleted** from the maximization problem.

Only term **(I)** matters: the firm maximizes over the states in which $p_t(j)$ is *stuck forever* (probability θ^s for each horizon s). The problem simplifies to:

$$\max_{p_t(j)} \quad \sum_{s=0}^{\infty} \theta^s\, E_t\left(\beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[\frac{p_t(j)}{P_{t+s}}\, y_{t+s}(j) - \phi_{t+s}\, y_{t+s}(j) \right] \right)$$

subject to the isoelastic demand constraint:

$$y_{t+s}(j) = \left(\frac{p_t(j)}{P_{t+s}} \right)^{-\epsilon} Y_{t+s}.$$

Substituting the demand constraint (and noting that by symmetry all re-optimizing firms choose the same price $p_t^* \equiv p_t(j)$):

$$\max_{p_t^*} \quad E_t \sum_{s=0}^{\infty} \theta^s \left(\beta^s \frac{\lambda_{t+s}}{\lambda_t} \right) \left[\left(\frac{p_t^*}{P_{t+s}} \right)^{1-\epsilon} Y_{t+s} - \phi_{t+s} \left(\frac{p_t^*}{P_{t+s}} \right)^{-\epsilon} Y_{t+s} \right]$$

(a) First-Order Condition

Taking the FOC with respect to p_t^*, using exponent $(1-\epsilon)$ on the revenue term and $(-\epsilon)$ on the cost term, then dividing through by $(1-\epsilon)$ and re-expressing via the demand constraint $y_{t+s}(j) = \left(\frac{p_t^*}{P_{t+s}}\right)^{-\epsilon} Y_{t+s}$:

$$\boxed{\sum_{s=0}^{\infty} \theta^s E_t\left[\beta^s \frac{\lambda_{t+s}}{\lambda_t}\left(\frac{p_t^*}{P_{t+s}} - \frac{\epsilon}{\epsilon-1}\,\phi_{t+s}\right) y_{t+s}(j)\right] = 0.} \tag{1}$$

Interpretation: The firm sets p_t^* so that, in probability-weighted present value across all states where this price remains stuck, marginal revenue $= \frac{\epsilon}{\epsilon-1}\phi_{t+s}$ (desired markup $\times$ marginal cost).

Implicit assumptions required for an interior solution:

1. *Sufficiently large markup* — ensures the firm always wants to produce.
2. *Small demand shock* — keeps the firm price-optimizing in period t (not at a corner).

2.2 Linearizing the FOC around the Zero-Inflation Steady State

Setup. Denote

$$\Lambda_{t+s} \equiv \beta^s \frac{\lambda_{t+s}}{\lambda_t}, \qquad Q_{t+s} \equiv \frac{p_t^*}{P_{t+s}}\, y_{t+s}(j) - \frac{\epsilon}{\epsilon-1}\,\phi_{t+s}\, y_{t+s}(j),$$

so the FOC reads $\sum_{s=0}^{\infty} \theta^s E_t[\Lambda_{t+s} Q_{t+s}] = 0$.

Steady-state values. In the zero-inflation steady state (denoted by bars): $p_t^* = P_{t+s} = P$, $\lambda_{t+s} = \lambda_t = \lambda$, $y_{t+s}(j) = y(j)$. Substituting:

$$\bar{Q} = \frac{P}{P}\, y(j) - \frac{\epsilon}{\epsilon-1}\,\bar{\phi}\, y(j) = y(j)\left(1 - \frac{\epsilon}{\epsilon-1}\,\bar{\phi}\right) = 0$$

$$\implies \quad \bar{\phi} = \frac{\epsilon-1}{\epsilon}.$$

(The steady-state real marginal cost equals the inverse of the desired markup.)

Linearization strategy. Because $\bar{Q} = 0$, a first-order approximation of the product $\Lambda_{t+s} Q_{t+s}$ around the steady state gives:

$$\Lambda_{t+s} Q_{t+s} \approx \bar{\Lambda}\, dQ_{t+s} + \underbrace{\bar{Q}}_{=0}\, d\Lambda_{t+s} = 1 \cdot dQ_{t+s}.$$

The $d\Lambda_{t+s}$ term drops out entirely.

Computing dQ_{t+s}. Differentiating Q_{t+s} with respect to its arguments:

$$dQ_{t+s} = \frac{1}{P_{t+s}}\, y(j)\, dp_t^* - \frac{p^*}{P^2}\, y(j)\, dP_{t+s} - \underbrace{\left(\frac{p^*}{P} - \frac{\epsilon}{\epsilon-1}\bar{\phi}\right)}_{=\,1-1\,=\,0} dy(j) - \frac{\epsilon}{\epsilon-1}\, y(j)\, d\phi_{t+s}.$$

The coefficient on $dy(j)$ vanishes because $\bar{p}^*/P = 1$ and $\frac{\epsilon}{\epsilon-1}\bar{\phi} = 1$. Thus:

$$dQ_{t+s} = \frac{y(j)}{P}\, dp_t^* - \frac{y(j)}{P}\, dP_{t+s} - \frac{\epsilon}{\epsilon-1}\, y(j)\, d\phi_{t+s}.$$

Switching to log-deviations (hat notation). Using $dp_t^* = P\,\hat{p}_t^*$, $dP_{t+s} = P\,\hat{P}_{t+s}$, and $d\phi_{t+s} = \bar{\phi}\,\hat{\phi}_{t+s}$ with $\frac{\epsilon}{\epsilon-1}\bar{\phi} = 1$:

$$dQ_{t+s} = y(j)\hat{p}_t^* - y(j)\hat{P}_{t+s} - y(j)\hat{\phi}_{t+s} = y(j)\left[\hat{p}_t^* - \hat{P}_{t+s} - \hat{\phi}_{t+s}\right].$$

Linearized FOC. Substituting back:

$$\sum_{s=0}^{\infty}(\theta\beta)^s \underbrace{y(j)}_{>0}\; E_t\left[\hat{p}_t^* - \hat{P}_{t+s} - \hat{\phi}_{t+s}\right] = 0.$$

Dividing by $y(j) > 0$ and separating $\hat{p}_t^*$ (which does not depend on s):

$$\hat{p}_t^* \sum_{s=0}^{\infty}(\theta\beta)^s = \sum_{s=0}^{\infty}(\theta\beta)^s E_t\left[\hat{P}_{t+s} + \hat{\phi}_{t+s}\right].$$

Since $\sum_{s=0}^{\infty}(\theta\beta)^s = \frac{1}{1-\theta\beta}$, multiplying both sides by $(1-\theta\beta)$:

$$\hat{p}_t^* = (1-\theta\beta)\sum_{s=0}^{\infty}(\theta\beta)^s E_t\left[\hat{P}_{t+s} + \hat{\phi}_{t+s}\right].$$

Recursive form. Split the $s = 0$ term from the rest:

$$\hat{p}_t^* = (1-\theta\beta)\left[\hat{P}_t + \hat{\phi}_t\right] + (1-\theta\beta)\sum_{s=1}^{\infty}(\theta\beta)^s E_t\left[\hat{P}_{t+s} + \hat{\phi}_{t+s}\right].$$

Re-indexing the second sum with $s' = s - 1$:

$$(1-\theta\beta)\sum_{s=1}^{\infty}(\theta\beta)^s E_t[\cdots] = \theta\beta\,(1-\theta\beta)\sum_{s'=0}^{\infty}(\theta\beta)^{s'} E_t[E_{t+1}[\hat{P}_{t+1+s'} + \hat{\phi}_{t+1+s'}]] = \theta\beta\, E_t\hat{p}_{t+1}^*.$$

Therefore:

$$\boxed{\hat{p}_t^* = (1-\theta\beta)\left[\hat{P}_t + \hat{\phi}_t\right] + \theta\beta\, E_t\hat{p}_{t+1}^*.} \tag{$*$}$$

Interpretation: The optimal reset price today is a weighted average of (i) the current price level plus current cost shock and (ii) what the firm would optimally choose tomorrow—reflecting the possibility that the price may be stuck.

2.3 Aggregate Price Dynamics

Starting from the aggregate price index $P_t^{1-\epsilon} = \int_0^1 p_t(j)^{1-\epsilon}\,dj$, partition the $[0,1]$ interval of firms according to who can and cannot reset prices in period t. Under Calvo pricing, each firm independently draws a "green light" with probability $(1-\theta)$, so:

$$P_t^{1-\epsilon} = \int_0^1 p_t(j)^{1-\epsilon}\,dj = \underbrace{\theta \int_0^1 p_{t-1}(j)^{1-\epsilon}\,dj}_{\text{fraction } \theta \text{ that cannot adjust}} + \underbrace{(1-\theta)(p_t^*)^{1-\epsilon}}_{\text{fraction } (1-\theta) \text{ that adjust to } p_t^*}.$$

Why is the first term valid? The Calvo lottery is i.i.d. and independent of each firm's current price level. Therefore, the cross-sectional distribution of prices among non-adjusters in period t is identical to the distribution across *all* firms in period $t-1$ (just scaled down by mass θ). By the law of large numbers:

$$\theta \int_0^1 p_{t-1}(j)^{1-\epsilon}\,dj = \theta P_{t-1}^{1-\epsilon}.$$

By symmetry, all adjusting firms face the same problem and choose the same reset price p_t^*, giving $(1-\theta)(p_t^*)^{1-\epsilon}$ for that group. This yields:

$$P_t^{1-\epsilon} = \theta\, P_{t-1}^{1-\epsilon} + (1-\theta)(p_t^*)^{1-\epsilon}. \tag{2}$$

Log-linearization of (2). Divide both sides by $P_{t-1}^{1-\epsilon}$ and define gross inflation $\Pi_t \equiv P_t/P_{t-1}$:

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\frac{p_t^*}{P_{t-1}}\right)^{1-\epsilon}.$$

Log-linearizing around the zero-inflation steady state $\Pi = 1$, $p_t^*/P_{t-1} = 1$:

$$\hat{\pi}_t = (1-\theta)\Big(\hat{p}_t^* - \hat{P}_{t-1}\Big). \tag{$**$}$$

2.4 Deriving the New Keynesian Phillips Curve (NKPC)

Combine $(*)$ and $(**)$. From $(**)$:

$$\hat{p}_t^* - \hat{P}_{t-1} = \frac{\hat{\pi}_t}{1-\theta} \quad\Longrightarrow\quad \hat{p}_t^* = \hat{P}_{t-1} + \frac{\hat{\pi}_t}{1-\theta}.$$

Substitute into $(*)$ and use $\hat{P}_t = \hat{P}_{t-1} + \hat{\pi}_t$:

$$\hat{P}_{t-1} + \frac{\hat{\pi}_t}{1-\theta} = (1-\theta\beta)\Big[\hat{P}_{t-1} + \hat{\pi}_t + \hat{\phi}_t\Big] + \theta\beta\left(\hat{P}_t + \frac{E_t\hat{\pi}_{t+1}}{1-\theta}\right).$$

After simplification (subtracting θP_{t-1} from both sides and collecting terms):

$$\frac{\hat{\pi}_t}{1-\theta} - \hat{\pi}_t = \theta\beta\frac{E_t\hat{\pi}_{t+1}}{1-\theta} + (1-\theta\beta)\hat{\phi}_t$$

$$\frac{\theta\,\hat{\pi}_t}{1-\theta} = \theta\beta\,\frac{E_t\hat{\pi}_{t+1}}{1-\theta} + (1-\theta\beta)\hat{\phi}_t$$

$$\hat{\pi}_t = \beta\,E_t\hat{\pi}_{t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta}\,\hat{\phi}_t.$$

Defining $\lambda \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}$ and noting that the (log deviation of) real marginal cost $\hat{\phi}_t = \widehat{mc}_t$:

$$\boxed{\hat{\pi}_t = \beta\,E_t\hat{\pi}_{t+1} + \lambda\,\widehat{mc}_t.} \tag{3}$$

2.5 Labor Market and Closing the Model

Aggregate labor demand. With production function $y_t(j) = A_t n_t(j)$, labor market clearing requires $N_t = \int_0^1 n_t(j)\,dj$. Aggregating:

$$y_t = A_t n_t + d_t \approx A_t n_t,$$

where d_t is the price dispersion term which is zero up to first order. In logs:

$$\hat{y}_t = \hat{a}_t + \hat{n}_t.$$

Real marginal cost. Using the household's labor supply condition $\hat{w}_t - \hat{P}_t = \sigma\hat{y}_t + \varphi\hat{n}_t$ and the firm's labor demand, the (log-deviation of) aggregate real marginal cost is:

$$\widehat{mc}_t = (\hat{w}_t - \hat{P}_t) - \widehat{mpn}_t = \sigma\hat{y}_t + \varphi\hat{n}_t - (\hat{a}_t - \hat{n}_t) = (\sigma+\varphi)\hat{y}_t - (1+\varphi)\hat{a}_t.$$

Natural rate of output. Under flexible prices, $\widehat{mc}_t = 0$ (markup is constant), so:

$$\hat{y}_t^n = \frac{1+\varphi}{\sigma+\varphi}\,\hat{a}_t.$$

Output gap. Define $\tilde{y}_t \equiv \hat{y}_t - \hat{y}_t^n$. Then:

$$\widehat{mc}_t = (\sigma+\varphi)\tilde{y}_t.$$

NKPC in terms of output gap. Substituting into (3):

$$\boxed{\hat{\pi}_t = \beta\,E_t\hat{\pi}_{t+1} + \kappa\,\tilde{y}_t, \qquad \kappa \equiv \lambda(\sigma+\varphi).} \tag{4}$$

3 Three-Equation New Keynesian Model

The complete system consists of:

1. **Dynamic IS (DIS):**

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma}(\hat{\imath}_t - E_t \hat{\pi}_{t+1} - \hat{r}_t^n), \qquad \hat{r}_t^n \equiv \rho + \sigma E_t \Delta \hat{y}_{t+1}^n.$$

2. **New Keynesian Phillips Curve (NKPC):**

$$\hat{\pi}_t = \beta \, E_t \hat{\pi}_{t+1} + \kappa \, \tilde{y}_t.$$

3. **Monetary Policy Rule (e.g., Taylor rule):**

$$\hat{\imath}_t = \phi_\pi \hat{\pi}_t + \phi_y \tilde{y}_t.$$