Graduate Macroeconomic Theory (ECN 200E) Discussion Section Week 6

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A model with investment adjustment costs

Decentralized RBC with Investment Adjustment Costs

Household problem:

$$V(k_t, Z_t, i_{t-1}) = \max_{c_t, N_t, i_t, k_{t+1}} \left\{ \log c_t + \frac{\theta}{1-\eta}(1-N_t)^{1-\eta} + \beta E_t V(k_{t+1}, Z_{t+1}, i_t) \right\}$$

subject to:

$$c_t + i_t = w_t N_t + r_t^k k_t + \pi_t$$

$$k_{t+1} = (1-\delta)k_t + \left(1 - \phi\left(\frac{i_t}{i_{t-1}}\right)\right) i_t$$

Household Lagrangian

$$\mathcal{L} = \log c_t + \frac{\theta}{1-\eta}(1-N_t)^{1-\eta} + \beta E_t V(k_{t+1}, Z_{t+1}, i_t)$$

$$+\lambda_t \left(w_t N_t + r_t^k k_t + \pi_t - c_t - i_t \right)$$

$$+\mu_t \left[(1-\delta)k_t + \left(1 - \phi\left(\frac{i_t}{i_{t-1}} \right) \right) i_t - k_{t+1} \right]$$

Multipliers:

$$\lambda_t = \text{marginal utility of wealth}$$

$$\mu_t = \text{shadow value of installed capital}$$

3. First Order Conditions

Consumption:

$$\frac{1}{c_t} = \lambda_t$$

Labor:

$$\theta(1 - N_t)^{-\eta} = \lambda_t w_t$$

Capital (Euler equation):

$$\mu_t = \beta E_t \left[\lambda_{t+1} \left(r^k_{t+1} + \frac{\mu_{t+1}}{\lambda_{t+1}}(1 - \delta) \right) \right]$$

3. First Order Conditions

Investment:

$$\lambda_t = \mu_t \frac{\partial k_{t+1}}{\partial i_t} + \beta E_t \left[\frac{\partial V_{t+1}}{\partial i_t} \right]$$

After some additional algebra (using envelope conditions):

$$\lambda_t = \mu_t \left[1 - \phi\left(\frac{i_t}{i_{t-1}}\right) - \phi'\left(\frac{i_t}{i_{t-1}}\right) \frac{i_t}{i_{t-1}} \right] + \beta E_t \left[\mu_{t+1} \phi'\left(\frac{i_{t+1}}{i_t}\right) \frac{i_{t+1}^2}{i_t^2} \right]$$

Tobin's q: Definition and Interpretation

Definition:

$$q_t = \frac{\mu_t}{\lambda_t}$$

- $\mu_t =$ shadow value of installed capital (utility units)
- $\lambda_t =$ marginal utility of consumption

Interpretation:

$$q_t = \text{Value of one more unit of capital in consumption units}$$

- If $q_t > 1$: Capital is valuable $\Rightarrow$ invest more
- If $q_t < 1$: Capital is not valuable $\Rightarrow$ invest less

Capital Euler Equation in Terms of q

$$\mu_t = \beta E_t \left[\lambda_{t+1} \left(r^k_{t+1} + q_{t+1}(1-\delta) \right) \right]$$

Divide by λ_t:

$$q_t = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(r^k_{t+1} + q_{t+1}(1-\delta) \right) \right]$$

Economic intuition:

Today's value of capital equals:

- discounted marginal product tomorrow
- plus discounted resale value of capital

Investment FOC in Terms of q

$$1 = q_t \left[1 - \phi\left(\frac{i_t}{i_{t-1}}\right) - \phi'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right] + \beta E_t\left[q_{t+1}\frac{\lambda_{t+1}}{\lambda_t}\phi'\left(\frac{i_{t+1}}{i_t}\right)\frac{i_{t+1}^2}{i_t^2}\right]$$

LHS: Marginal cost of one unit of investment

RHS: Marginal benefit

- First term:

$$q_t \times (\text{effective capital created today})$$

- Second term: Future adjustment cost effects

$$\beta E_t[q_{t+1} \times \text{effect on future costs}]$$

Key intuition:
Investment decision balances:

$$\text{Cost today} = \text{Value of installed capital} + \text{Impact on future adjustment costs}$$

A Preview: The Stochastic Discount Factor (SDF)

Definition:

$$m_{t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t}$$

Asset pricing condition:

$$1 = E_t \left[m_{t+1} R_{t+1} \right]$$

Economic meaning:

- It prices all assets in equilibrium
- It is high in **bad times tomorrow** (when marginal utility is high)
- Connects macro models to finance
- Explains risk premia via covariance:

$$E[R] \uparrow \quad \text{if} \quad \mathrm{Cov}(m_{t+1}, R_{t+1}) < 0$$

- In RBC, SDF = discounted intertemporal MRS

Sample Question

Macro Prelim 2019 Q3