

Graduate Macroeconomic Theory (ECN 200E)

Discussion Section Week 4

Jinho Kim

UC Davis

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Balanced Growth Path: Deriving Results

Step 1 — Show $\gamma_Y = \gamma_C = \gamma_I$

Want to show: $\gamma_Y = \gamma_K = \gamma_C = \gamma_I = \gamma_X$

Assumption: Y_t, C_t, I_t, K_t grow at constant rates. (time-invariant), N_t is bounded

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where $c_t = C_t/Y_t$, $i_t = I_t/Y_t$.

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where $c_t = C_t/Y_t$, $i_t = I_t/Y_t$.

Suppose $\gamma_C \neq \gamma_I$. First, assume $\gamma_C > \gamma_I$:

$$\begin{aligned}\gamma_Y &= \gamma_C c_t + \gamma_I (1 - c_t) \\ \Rightarrow \gamma_Y - \gamma_I &= (\gamma_C - \gamma_I) c_t > 0\end{aligned}$$

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Continuation: Assume $\gamma_C > \gamma_I$, and we found $\gamma_Y > \gamma_I$.

- Then C_t grows faster than $Y_t \Rightarrow c_t = C_t/Y_t$ increases over time.
- But this contradicts the **BGP assumption**: c_t must be constant.

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$$\Rightarrow \gamma_C = \gamma_I$$

Then, from (1):

$$\begin{aligned}\gamma_Y &= \gamma_C c_t + \gamma_C (1 - c_t) = \gamma_C (c_t + 1 - c_t) = \gamma_C \\ \Rightarrow \gamma_Y &= \gamma_C = \gamma_I\end{aligned}$$

Conclusion: All aggregate quantities grow at the same rate on the BGP.

Step 2: $\gamma_Y = \gamma_K$

Consumption-Investment share is constant:

$$C_t = s_c Y_t, \quad I_t = (1 - s_c) Y_t \quad \text{for constant } s_c$$

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$$C_t = s_c Y_t, \quad I_t = (1 - s_c) Y_t \quad \text{for constant } s_c$$

Law of motion of capital:

$$\begin{aligned} K_{t+1} &= (1 - \delta) K_t + I_t \\ \Rightarrow \frac{K_{t+1}}{K_t} &= (1 - \delta) + \frac{I_t}{K_t} \\ \Rightarrow \gamma_K &= (1 - \delta) + \frac{(1 - s_c) Y_t}{K_t} \end{aligned}$$

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$$\Rightarrow \frac{Y_t}{K_t} = \underbrace{(\gamma_K - (1 - \delta))}_{\text{RHS are all constants}} \cdot \frac{1}{1 - s_c}$$

Therefore: Y_t/K_t constant $\Leftrightarrow \gamma_Y = \gamma_K$

Step 3: $\gamma_Y = \gamma_X$

Cobb-Douglas:

$$Y_t = A_t K_t^\alpha (X_t N_t)^{1-\alpha}$$

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Divide both sides by Y_t :

$$\frac{Y_{t+1}}{Y_t} = \underbrace{\left(\frac{A_{t+1}}{A_t}\right)}_{\text{No trend} = 1} \left(\frac{K_{t+1}}{K_t}\right)^\alpha \underbrace{\left(\frac{L_{t+1}}{L_t}\right)^{1-\alpha}}_{N_t^s \text{ is bounded, } \gamma_L = 1} \left(\frac{X_{t+1}}{X_t}\right)^{1-\alpha}$$

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Taking logs:

$$\log \gamma_Y = \alpha \log \gamma_K + (1 - \alpha) \log \gamma_X$$

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But from earlier, $\gamma_Y = \gamma_K$.

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But from earlier, $\gamma_Y = \gamma_K$.

$$\Rightarrow \gamma_Y = \gamma_X$$

Conclusion: Growth rates are all equal and constant:

$$\gamma_Y = \gamma_K = \gamma_C = \gamma_I = \gamma_X$$

Balanced Growth Path: Sample Question

Midterm Question 2: RBC Model with Trend Growth

- Representative agent with preferences over consumption and labor:

$$\begin{aligned} \max_{\{C_t, N_t, K_{t+1}\}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \psi_t \frac{N_t^{1+\kappa}}{1+\kappa} \right) \\ \text{s.t.} \quad & C_t = w_t N_t, \\ & K_{t+1} = (1-\delta)K_t + I_t, \\ & Y_t = a_t K_t^\alpha (X_t N_t)^{1-\alpha} \\ & \gamma_Y = \gamma_C = \gamma_K = \gamma_X > 1, a_t = 1 \end{aligned}$$

- (a) Are the preferences with constant $\psi_t = \bar{\psi}$ consistent with constant labor supply?
- (b) What changes when $\psi_t = \bar{\psi} X_t^{1-\sigma}$? Show using algebra and intuition.
- (c) Derive growth rates of w_t and r_t^k under BGP with $\psi_t = \bar{\psi} X_t^{1-\sigma}$; compare to data.

Overview of the Question

- We analyze how preferences interact with long-run growth to determine labor supply behavior.
- Focus: Can the model generate constant hours worked along a balanced growth path?
- We compare two cases: constant disutility scale vs. growth-adjusted disutility scale.

Big Picture: What's This Really About?

Stylized Facts from the Data

- **Growth:** Real aggregate variables (output, consumption, capital) grow steadily over time.
 - **Common trends:** Permanent technology shocks generate common long-run effects across macro variables.
 - **Stable factor shares:** Labor and capital income shares remain relatively stable over time.
 - **Divergent labor patterns:** Real wages have increased substantially, while hours worked per worker have remained roughly constant.
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- Question: How can we write preferences so that labor supply does NOT grow with income?
 - This is crucial for generating a balanced growth path consistent with empirical patterns.

Factor Prices Under BGP

Firm's problem:

$$\max_{K_t, N_t} a_t K_t^\alpha (X_t N_t)^{1-\alpha} - w_t N_t - r_{k,t} K_t$$

First-order conditions:

$$w_t = (1 - \alpha) a_t K_t^\alpha X_t^{1-\alpha} N_t^{-\alpha}$$

$$r_{k,t} = \alpha a_t K_t^{\alpha-1} (X_t N_t)^{1-\alpha}$$

Growth Rates of Prices

From BGP conditions:

- If $\gamma_Y = \gamma_K$ and $\gamma_N = 1$:

$$\gamma_w = \frac{\gamma_X}{\gamma_N} = \gamma_X$$

- Rental rate:

$$\gamma_r = \frac{\gamma_X}{\gamma_K} = 1$$

- Interpretation: real wages grow, rental rate stays constant

When Do We Use a Static vs. Dynamic Approach?

- If the question asks about **long-run patterns** (e.g., whether N_t stays constant along the balanced growth path), it often suffices to use a **static intratemporal condition**.
- Static methods focus on **within-period trade-offs** (e.g., substitution vs. income effect).
 - Capital income (e.g. $r_{k,t}K_t$) is predetermined and acts like an endowment.
 - Examine how N_t responds to changes in w_t , holding everything else constant.
- We care only about whether the **equilibrium level of N_t grows** over time — a comparative statics result $\Rightarrow$ a one-period (static) analysis is sufficient

(a) Constant $\Psi_t = \bar{\Psi}$

Setup:

$$\begin{aligned} \max_{C_t, N_t} \quad & \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \bar{\Psi} \cdot \frac{N_t^{1+\kappa}}{1+\kappa} \\ \text{s.t.} \quad & C_t = w_t N_t \end{aligned}$$

Lagrangian:

$$\mathcal{L} = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \bar{\Psi} \cdot \frac{N_t^{1+\kappa}}{1+\kappa} + \lambda_t (w_t N_t - C_t)$$

First-order conditions:

$$C_t^{-\sigma} = \lambda_t$$

$$\bar{\Psi} N_t^\kappa = w_t \lambda_t \Rightarrow \bar{\Psi} N_t^\kappa = \frac{w_t}{C_t^\sigma}$$

Substitute budget constraint:

$$N_t = \left(\frac{1}{\bar{\Psi}} w_t^{1-\sigma} \right)^{\frac{1}{\kappa+\sigma}}$$

(a) Interpretation: Why This Fails

- In equilibrium, the real wage equals the marginal product of labor:

$$w_t = (1 - \alpha) \frac{Y_t}{N_t}.$$

- Along a balanced growth path (BGP), output grows due to technological progress:

$$Y_t \propto X_t. \Rightarrow w_t \propto X_t$$

- From the labor supply condition with constant $\Psi_t = \bar{\Psi}$,

$$N_t = \left(\frac{1}{\bar{\Psi}} w_t^{1-\sigma} \right)^{\frac{1}{\kappa+\sigma}},$$

higher wages imply higher labor supply: **intratemporal substitution effect**

- Therefore, as wages grow along the BGP, hours worked also grow:

$$w_t \uparrow \Rightarrow N_t \uparrow.$$

- This contradicts the empirical fact that hours per worker are roughly constant over time.

(b) Growth-Adjusted Disutility: $\Psi_t = \bar{\Psi} X_t^{1-\sigma}$

Modified FOC:

$$\Psi_t N_t^\kappa = \frac{w_t}{C_t^\sigma} \Rightarrow \bar{\Psi} X_t^{1-\sigma} N_t^\kappa = \frac{w_t}{C_t^\sigma}$$

Rewriting:

$$N_t = \left[\frac{1}{\bar{\Psi}} \left(\frac{C_t}{X_t} \right)^{1-\sigma} \right]^{\frac{1}{\kappa+1}}$$

Key idea: If $\frac{C_t}{X_t}$ is constant $\Rightarrow N_t$ is constant.

$$C_t \propto Y_t \propto X_t \Rightarrow \frac{C_t}{X_t}$$

(b) Why $C_t \propto X_t$ Implies Constant N_t

Economic intuition:

- As the economy grows, both C_t and w_t increase with X_t .
- Disutility from labor Ψ_t also grows proportionally (since $\Psi_t = \bar{\Psi} X_t^{1-\sigma}$).
- This scaling offsets the substitution effect: households are richer, but they don't increase hours worked.
- So, N_t remains constant *despite* growth in income and wages.

Final Interpretation

- Case (a): constant Ψ_t leads to increasing hours $\Rightarrow$ contradicts data
- Case (b): scaling Ψ_t with $X_t^{1-\sigma}$ leads to constant hours $\Rightarrow$ matches data
- Real wages grow over time $\Rightarrow$ consistent with $\gamma_w = \gamma_X$
- Rental rate of capital stable $\Rightarrow$ consistent with $\gamma_r = 1$

Labor Supply: Intuition

Labor Supply: Substitution vs. Wealth Effect

Key Trade-off in Labor Supply Decisions

- Households choose hours N_t to balance:
 - **Substitution Effect:** Higher $w_t \Rightarrow$ work more (labor is more valuable)
 - **Wealth Effect:** Higher w_t or productivity $\Rightarrow$ consume more leisure (work less)

Substitution Effect:

- Real wage w_t increases the opportunity cost of leisure rises
- Households are incentivized to substitute away from leisure toward labor
- $\uparrow w_t \quad \uparrow N_t$

Wealth Effect:

- Real wage w_t or productivity increases lifetime income rises
- Households feel richer and demand more leisure (a normal good)
- $\uparrow w_t \quad \downarrow N_t$

Which Effect Dominates?

Depends on Preferences

- **Utility curvature:**
 - How strongly utility increases in C vs. disutility in N
 - e.g., CRRA utility: σ , labor disutility: η, ε
- **Frisch Elasticity of Labor Supply:**
 - Higher elasticity $\Rightarrow$ labor supply more responsive to wage
 - Substitution effect tends to dominate

Depends on Timing Horizon

- **Short-run:** Substitution effect dominates
- **Long-run:** Wealth effect may reduce labor supply

Depends on Shock Persistence

- **Temporary TFP shock:** Substitution effect stronger
- **Persistent TFP shock:** Large wealth gains $\Rightarrow$ wealth effect stronger

Labor Supply: Sample Question

Sample Question

Compare labor supply responses under different labor supply formulations. The household's intratemporal first-order condition gives labor supply as:

$$\eta N_t^{\eta-1} = w_t \quad (\text{when } \varepsilon = 0) \quad (1)$$

$$\eta N_t^{\eta-1} \cdot \frac{1 - N_t^\eta}{\lambda_t} = w_t \quad (\text{when } \varepsilon = 1) \quad (2)$$

Assume $\lambda_t = \frac{1}{C_t}$. You do not need to derive these conditions. Take them as given.

Questions:

- (a) Linearize the labor supply conditions. (8 marks)
- (b) Derive the Frisch elasticity of labor supply in each case. **If both are calibrated to the same elasticity**, which generates a larger labor response to a TFP shock? (10 marks)

(a) Linearize Labor Supply Conditions

Case 1: $\varepsilon = 0$

$$\eta N_t^{\eta-1} = w_t$$

$$\ln \eta + (\eta - 1) \ln N_t = \ln w_t$$

$$(\eta - 1) \hat{N}_t = \hat{w}_t$$

$$\Rightarrow \hat{N}_t = \frac{1}{\eta - 1} \hat{w}_t$$

(a) Linearize Labor Supply Conditions

Case 2: $\varepsilon = 1$

$$\eta N_t^{\eta-1} \cdot \frac{1 - N_t^\eta}{\lambda_t} = w_t \quad (\text{Let } B_t \equiv 1 - N_t^\eta)$$

$$\eta N_t^{\eta-1} B_t = \lambda_t w_t$$

$$\ln \eta + (\eta - 1) \ln N_t + \ln B_t = \ln \lambda_t + \ln w_t$$

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$$\ln \eta + (\eta - 1) \ln N_t + \ln B_t = \ln \lambda_t + \ln w_t$$

We can derive:

$$\hat{B}_t = -\frac{\eta N_t^\eta}{1 - N_t^\eta} \hat{N}_t$$

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We can derive:

$$\hat{B}_t = -\frac{\eta N_t^\eta}{1 - N_t^\eta} \hat{N}_t$$

Linearize the main equation:

$$\left(\frac{\eta - 1 + N_t^\eta}{1 - N_t^\eta} \right) \hat{N}_t = \hat{w}_t + \hat{\lambda}_t$$

$$\Rightarrow \hat{N}_t = \frac{1 - N_t^\eta}{\eta - 1 + N_t^\eta} (\hat{w}_t + \hat{\lambda}_t)$$

(b) Frisch Elasticity TFP Shock Response

Compare Wage Elasticity of Labor Supply

- Assume both models calibrated to same Frisch elasticity.
- Case $\varepsilon = 0$: $\hat{N}_t = \frac{1}{\eta-1} \hat{w}_t$
- Case $\varepsilon = 1$: $\hat{N}_t = \frac{1-N_t^\eta}{\eta-1+N_t^\eta} (\hat{w}_t + \hat{\lambda}_t)$
- $\hat{\lambda}_t < 0$ when c_t increases after TFP shock (since $\lambda_t = 1/C_t$)
- **So:** wage response is muted due to negative $\hat{\lambda}_t$

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- Case $\varepsilon = 1$: $\hat{N}_t = \frac{1-N_t^\eta}{\eta-1+N_t^\eta} (\hat{w}_t + \hat{\lambda}_t)$
- $\hat{\lambda}_t < 0$ when c_t increases after TFP shock (since $\lambda_t = 1/C_t$)
- **So:** wage response is muted due to negative $\hat{\lambda}_t$

Final Answer:

- Model with $\varepsilon = 0$ generates **larger hours response** to TFP shock.
- In $\varepsilon = 1$, rising consumption lowers marginal utility offsets substitution effect.